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Intervention in Textual Comprehension of Mathematical Word Problems: A Case Study

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Abstract

Word Mathematical Problem solving involves a phase of Comprehension or construction of a Situational Model. This paper presents an intervention with a high school student who presented difficulties in the construction of situational models in Word Mathematical Problem Solving. A qualitative research with an instrumental case study design was developed to obtain a profile of the type of difficulties and to intervene by means of semi-structured interviews. The role of situation drawing and the use of concrete materials during the interviews as a strategy to encourage the construction of situation models is also reported.

Keywords: Textual Comprehension, Difficulty, Situation Model, Word Problem, Secondary

Introduction

Mathematical Word Problem is relevant from a didactic and research point of view. In Mathematics Education it is considered a didactic strategy to "test the acquisition of concepts or ideas, or the function of algorithms and their use" (D'Amore et al., 1996, p. 56). It is a learning experience that brings students closer to mathematical knowledge (Puig & Cerdán, 1988), to use it in a creative and transferable way to other contexts (Santos-Trigo, 1999).

Studies on Mathematical Word Problem Solving have led researchers to explore the components and phases involved (Polya, 1995; Puig & Cerdán, 1988; Leiss, Schukajlow, Blum, Messner & Pekrun, 2010; Cummins, Kinstch, Reusser, & Weimmer, 1988; Rellensmann, Schukajlow & Leopold, 2017). Studies about Cognition point out that the Situation Model is the mental representation of the actions, events and properties of a protagonist in a space and time determined by the text. Explicit and implicit information constitute such a model (van dijk & Kintsch, 1983; Therriault & Rinck, 2007; Zwaan & Radvansky, 1998; Juarez, Mejia, Gonzalez, & Slisko, 2014; Rellensmann

et al., 2017). This construct has been used to explain Textual Comprehension processes, as well as to analyze difficulties in Word Problem solving in basic education students (Leiss, et al., 2010; Juárez, Slisko, Hernández, & Monroy, 2015). Difficulties have been identified that persist across the time (Wijaya, Retnawati, Setyaningrum, Aoyama, & Sugiman, 2019) and become "barriers that prevent access and participation in mathematics learning activities" (Chin & Fu, 2021 p. 532).

The aim of this paper is to analyze the difficulties of a high school student in the Comprehension phase of Word Problems. We employ as a theoretical foundation the Event Indexing Model (Zwaan, Langston & Graesser, 1995), a qualitative research approach and an instrumental case study (Stake, 1999). The research question is What are the difficulties in the phase of constructing situation models during the resolution of Word Problems? How can they be addressed in order to promote Textual Comprehension?

Background

Studies on Mathematical Word Problem Solving have led researchers to explore the components and phases involved (Polya, 1995; D'Amore et al.,1996; Puig and Cerdán, 1988; Leiss et al., 2010; Cummins et al., 1988; Rellensmann et al., 2017). D'Amore et al. (1996) provide a general definition of a School Mathematical Problem as "a description of a context (often a realistic context referring to actual, assumed, or potential facts), some numerical data, and an explicit question" (p. 53). Word mathematical Problem is a particular type of School Mathematical Problem.

Puig and Cerdán, 1988 refers to the arithmetic component of Word Problems. They state that in the resolution it is important to identify the numerical data and the arithmetic operations to be performed. However, Word Problems also include a narrative component (Zan, 2011). This component is useful to build a mental image of the situation. Cognitive approach studies identify the Situation Model as a primary component (van dijk & Kintsch, 1983; Therriault & Rinck, 2007; Zwaan & Radvansky, 1998; Juarez et al., 2014; Rellensmann et al., 2017).

Other researchers point out the phases of reading, comprehension, translation and calculation in Word Problem Solving (Puig & Cerdán, 1988). Polya points out that reading and Comprehension are linked and initial aspects of the process. In the translation phase, or, "elaboration of a plan", one moves from verbal expression to an arithmetic expression of the problem. In the calculation phase, or phase for the "execution of the plan", the arithmetic operations that are deemed appropriate are applied. Zan, 2011, classifies these processes into a Construction Phase and a Solution Phase. The first demands linguistic skills for Textual Comprehension, and the latter requires mathematical skills (Leiss et al., 2010; Cummins et al., 1988). Adequate performance in

the Solution Phase depends on adequate performance in the first phase (D'Amore et al., 1996; Zan, 2011; Verschaffel, Greer, & De Corte).

Going through the different phases of the problem-solving process consolidates the learning and production of mathematical knowledge, however, students with lower mathematical abilities (Rellensmann et al., 2017), come to encounter difficulties or obstacles. Weakly developed linguistic skills generate difficulties in verbal problem solving. One difficulty that arises is the suspension of the construction of meaning (Verschaffel et al., 2000). This phenomenon occurs when the student omits the construction of an adequate representation of the problem situation, focusing instead on finding quantities and numbers that relate to the question. Traditional school teaching practices based on the mechanical resolution of the problems implicitly influence this phenomenon (D'Amore et al., 1996). Even the length, the realistic or unrealistic context of the problem statement and the question posed are variables that influence resolution (Puig and Cerdán, 1988; Gerofsky, 1996; Verschaffel et al., 2000; Zan, 2011; Juárez et al., 2015).

One factor that has been studied and that affects Word Problem Solving is narrative break. According to Zan (2011) this occurs when "the question and the information needed for the solution are not consistent from the point of view of the narrated story" (p. 53). Students may generate answers with an apparent suspension of sense-making. For example, in the problem the age of the captain: "There are 26 sheep and 10 goats on a ship. How old is the captain?", a student gives as an answer 37 and adds that perhaps the captain receives an animal as a gift for each birthday (IREM de Grenoble, 1980). Faced with the apparently absurd response, Zan (2011) explains this result as the lack of "possible formal connections" between narrative thinking and logical or procedural thinking (Bruner, 1986, p. 24). When a narrative break occurs, the responses generated are more oriented to give unity and justifications to the narrative, while the operations are consequently arbitrary and more adjusted to the narrative than to logical thinking. Therefore, it highlights the importance of the narrative of the problem for the question to make sense.

Conceptual Framework

The Situation Model is a mental model constructed by readers who achieve comprehension of a statement or a written text. Text comprehenders construct mental representations of the situations described in the text (Leiss et al., 2010; Jhonson-Laird, 1983; van Dijk & Kintsch, 1983). During Comprehension, the reader brings to mind knowledge from everyday life, those memories generated by interactions with the environment (Therriault & Rinck, 2007). These information stores or schemas are used to generate interpretations during Textual Comprehension; determine the essential parts and

infer information (Zwaan & Radvansky,1998; Silva, 2014). In the creation of a Situation Model, schemas are fundamental elements (Zwaan & Radvansky, 1998), constituted by the evaluations of the narrated situation and the affective impression it provokes in the reader (D'Amore, 2006).

In the Event Indexing Model, precisely the events or "vicissitudes of a protagonist's intention" (Bruner, 1986, p. 29) can be linked to one or more dimensions through indexes. It is worth asking what makes a Text Comprehender construct an appropriate indexing between events and dimensions? From this theoretical perspective, the answer refers to a property of information called relevance. This is "information that is foregrounded by creating and maintaining a retrieval cue [...] in short-term memory" (Zwaan & Radvansky, 1998, p.167). The Comprehension is the result of the reader's tracking of the relevant elements of each dimension (Protagonist, Intentionality, Spatiality, Temporality and Causality) for the construction of a complete Situation Model. Based on the information processing framework for the construction of the *SM* (Situation Model), the following relevant elements were identified for each Dimension:

In the Protagonist Dimension the relevant element is to identify the tokens, or protagonist of the situation. During reading, the reader constructs in his or her mind tokens (subjects or objects of the story). These tokens have goals and some immediate or intuitively perceivable intentionality for the reader (Bruner, 1986). Thus, the Protagonist dimension is closely related to the intentionality dimension. The latter involves the tracking of the protagonist's achieved or unfulfilled goals.

In the Spatiality Dimension, the most representative relevant elements are the description of a space determined by the context of the situation, the description of facts that clearly refer to the same situation and the establishment of properties to establish the relationship between the protagonist, the objects and the space. For our study we assume that numbers fulfill this property. The quantities described or implied in the text of the problem establish the spatial relationship between the protagonist and the objects.

The Temporal Proximity of events is a relevant element of the Temporality Dimension, which can be identified by means of temporal markers (Zwaan & Radvansky, 1998). Finally, in the Causality Dimension the relevant element is some causal relationship explicit or inferred by the reader by means of his schemata.

Method

This work adopted the qualitative research approach and the instrumental case study (Stake, 1999).

Choice of Case

A sampling technique with a defined purpose was developed (Hernández-

Sampieri, 2006) to identify a secondary school student with difficulties in the Comprehension of Word Problems. The study was developed in a secndary school in the city of Puebla, Mexico.

Materials

For data collection were used worksheets and semi-structured interviews with the use of specific materials.

Worksheets

In the worksheets, three verbal problems (PV1, PV2 and PV3) and boxes for the respective drawings were posed. The previous indication to the problems was to read carefully the situations and answer the questions. After each problem, drawings were requested. Table 1 shows the problems used.

Table 1: Word problems

PV1: A soldier guards a wall. The wall has a gate in its center. The soldier was 16 meters to the left of the gate when he heard a noise coming from the right side of the wall. He walked 35 meters to the right and stopped when he realized that it had been a false alarm. At that moment he decided to sit down to rest, he looked towards the door. How many meters from the gate is the soldier? He Is he to the right or to the left of the door?
Bruno and Espinel (2002)
PV2: A bowler is five steps behind the throwing line, at that moment the distance between him and the pins is 21 m. The player advances through the distance available to him before delivering the ball. If the track where it slides has a length of 18 m: What is the distance traveled by the player before throwing the ball? What is the length of one step of the player?
Own elaboration
PV3: Pablo jumps from a springboard and rises 1m in the air, falls 5 meters into the water and then goes up 2 meters to reach the surface of the water. How high is the diving board above the water level?
Islas et al. (2012)

We base the choice of these problems considering six qualitative variables of the task, that is, variables “that have to do exclusively with the problem” (Puig & Cerdán,

1988, p.17). We name these narrative features of word problems:

Presence of a protagonist (1): referred to a character or agent (person or object that produces an action), Intention of the protagonist (2): referred to the objectives and goals of the protagonist, "it is immediately and intuitively recognizable" (Bruner, 1986, p.29). Presence of some causal relationship (3): "when objects move with respect to each other, within very limited conditions, we see causality" (Bruner, 1986, p. 29). Presence of spatial-temporal relationships (4): crucial dimension to establish the relationship between the objects and/or protagonist (Zwaan and Radvansky, 1998) through directionality (up, down, forward, backward, left and right). Presence of a conflict (5): referring to the events that represent some "vicissitude of the intention" in the story (Bruner, 1986, p. 29), and finally, the location of the question in the statement of the problem (6): the question can be located at the beginning of the text, at the end of the text, or the complete text constitutes the question (Puig and Cerdán, 1988). Table 2 represents the analysis of these variables for each verbal problem.

Table 2: Narrative characteristics of word problems

Variable PV	Protagonist	Intention	Causality	Spatial-temporal relations	Conflict	Question location
PV1	Soldier	guard a wall	The soldier is displaced as a result of another event	Left-right implied directionality	The soldier heard a noise coming from the opposite side of his starting position.	In the end
PV2	Player	bowling	The player moves only the available distance	Directionality implied front-back	The player does not immediately release the ball from its initial position.	In the end
PV3	Pablo	perform a dive	Pablo moves right away from another event	Directionality implied up-down	The diver does not immediately launch into the pool from his starting position.	In the end

Source: Self made

The previous analysis allowed us to point out the absence of a narrative break, and a good story with relevant elements that are assumed to be sufficient for

understanding.

Semi-Structured Interviews

Three questionnaires were elaborated to analyze the three Word Problems respectively. They were elaborated as follows: the Dimensions of the SM (1) Protagonist, (2) Intentionality, (3) Spatiality, (4) Temporality and (5) Causality were considered thematic areas, initiated with an open question and ended with a question of distension for the interviewee. Each questionnaire consisted of five dimensions, one indicator for each dimension, except in the Spatial Dimension, from which three indicators were broken down, and on average three items were generated for each indicator, as shown in Table 3.

Table 3: Dimensions, indicators, and items

Dimension	Indicator	Items
Protagonist	Identification of tokens or protagonist	1. What objects and characters are mentioned in the problem? 2. Who is the protagonist? 3. Do you consider this information important to understand the problem?
Intentionality	Identification of the objective or objectives achieved or unsatisfied of the protagonist	4. What was the soldier's goal? 5. Do you think it is important to know the main character's intention or objective in order to understand the problem?
Spatiality	Description of a space determined by the context of the situation	6. Could you describe the position of the objects mentioned in the problem? 7. What was the initial distance of the soldier from the door? 8. How many meters did the soldier move from where he was initially? 9. In which direction did the soldier move? 10. What do the numbers written in the problem represent? 11. What was the final position of the soldier with respect to the door? 12. Does the information about the space in which the situation occurred seem important to you to understand the problem?
Temporality	Description of	13. If you had to order the actions that happened

	facts that refer to the same situation.	chronologically, how many actions would you count? 14. At what point in the situation is it narrated: "A soldier guards the wall": at the beginning, in the middle or at the end? 15. Do you consider it important to know what happened before and what happened after to understand the problem?
Causality	Establishment of properties (numbers) to establish spatial relationships between objects and/or protagonist	16. What generated the displacement of the soldier when he was in his initial position? 17. Why did the soldier stop when he was walking to the right? 18. Do you consider this information important to understand the problem?

Source: self-made

The items in Table 3 made up questionnaire 1. Questionnaires 2 and 3 retain the same design (dimensions and indicators).

Procedure

Data collection from the worksheets generated primary responses and drawings (pre-interview), as well as final responses and drawings (post-interview). According to the research design, we relied on primary data to identify the most useful and convenient case (Stake, 1999). Thus, we identified Carlos (pseudonym) who provided incorrect answers and few elements represented in the drawings.

The interviews included the development of the rapport, the initial worksheet was handed out and participation was requested to carry out the interviews. The confidentiality and protection of the data and the non-affectation of their qualifications were explained. At the end of the interview, the initial worksheet was collected and the problem was delivered again on a new worksheet to obtain final answers and drawings. The materials that represented the objects and protagonists (tokens) mentioned in the problems (the soldier, the wall, the trampoline, the bowling game, etc.) were also given to Carlos. These materials are shown in Figure 1.

Figure 1: Specific materials used in interviews

Carlos solved each PV on new worksheet and made the corresponding drawings. The daily interviews lasted an average of 50 minutes each one. The interviews were videotaped. These and the drawings were analyzed to categorize the responses.

Analysis Strategy

The data from the interview transcripts and drawings generated by Carlos were analyzed using the thematic analysis proposed by Clarke and Braun (2013). Four steps were developed according to the thematic analysis model. In the first step, the videotaped interviews and Carlos's drawings were analytically observed. In the second step: the most significant data from the interview transcripts and drawings were identified and coded to answer the study's research question. In the third step, all the coded and significant data for the construction of the themes were collected. In the fourth step: congruence between the themes and the coded extracts and the complete data set was checked.

Analysis Categories

For this study, three categories of analysis were defined, grouped into two large themes. The topics are: (1) Difficulties in the construction of SM and (2) Characteristics of the drawings. The first theme was analyzed from the category: follow-up of relevant elements of the SMs. In the second theme, two categories were established: situational information of the drawings (complete, partial, or inconsistent) and quality of the response (correct or incorrect). Each drawing and response were categorized based on the relevant elements observed in them and through a dichotomous and observation procedure. Table 4 presents the organization of the analysis topics, their respective categories, indicators, and instruments.

Table 4: Analysis Categories (Rellensmann et al., 2017; Zwaan & Radvansky, 1998)

Issue	Category	Indicator	Instrument
Difficulties in the construction of SM	Follow-up of relevant elements of the SMs	Tracking of relevant elements (chips or tokens, objectives, description and spatial relationship and temporal proximity) during reading and solving verbal problems	interview questionnaires
Characteristics of the drawings	Situational information of the drawings	Representation of relevant elements (chips or tokens, objectives, description and spatial relationship and temporal proximity) according to the Event Indexing Model.	Worksheet (drawings section)
	Response quality	Ranking of responses with respect to a correct or incorrect solution	Worksheet (answers section)

Source: self-made

Analysis Tools

The analysis tools were developed by the authors. The analysis of the interview transcripts was carried out through an Observation Scale. The codes DP, DI, DE, DT, DC, correspond to the initials of each of the dimensions of the situation models: the letter D corresponds to the initial of the word Dimension and P, I, E, T, C are the initials of Protagonist, Intentionality, Spatiality, Temporality and Causality respectively.

In total, seven indicators were considered. One indicator for each Dimension (DP, DI, DT, and DC), except for DT: which has three indicators. All indicators were labeled with lowercase letters in alphabetical order, for example, the PD indicator was assigned the letter a, the ID indicator was assigned the letter b and so on until DC with the indicator marked with the letter g.

Items were numbered from 1 to 18. The relationship between Dimensions, Indicators, and Items was labeled as follows: initials of the Dimension in uppercase, letters in lowercase alphabetical order for the indicator, and item number. In the following lines we explain two examples, item 1 attends to the indicator of the Protagonist Dimension, this relationship was labeled as DPa1. Item 10 attends to the third indicator of the Spatiality Dimension, therefore, it corresponds to DEe10.

It should be noted that the closed questions asked at the end of each dimension were raised as questions of relaxation and to mark the passage to the next dimension. Despite being included in the questionnaire, questions related to the monitoring of relevant elements are not considered. These are the items marked with the codes DPa3, DIb5, DEe12, DTf15 and DCg18. The same coding was carried out with the

questionnaires to analyze PV2 and PV3. Said Observation Scale is shown in Table 5.

Table 5. Observation scale

Purpose: Identify the follow-up of relevant elements of the SM in the reading of word problems				
Indicator	Follow-up			
Author(s)	Yes	No	Partially	Does not answer
DPa Identification of tokens or protagonist				
DPa1 What objects and characters are mentioned in the problem? DPa2 Who is the protagonist? DPa3 Do you consider it important to identify the protagonist to understand the problem?				
DIb Identification of the objective or objectives achieved or unsatisfied of the protagonist				
Dib4 What was the soldier's goal? DIb5 Do you consider it important to know the main character's intention or objective in order to understand the problem?				
DEc Description of a space determined by the context of the situation				
DEc6 Could you describe the position of the objects mentioned in the problem? DEc7 What was the initial distance of the soldier from the door?				
DEd Description of facts that refer to the same situation.				
DEd8 How many meters did the soldier move from where he was initially? DEd9 In which direction did the soldier move?				
DEe Establishment of properties (numbers) to establish spatial relationships between objects and/or protagonist				
DEe10 What do the numbers written in the problem represent? DEe11 What was the final position of the soldier with respect to the door? DEe12 Does the information about the space in which the situation occurred seem important to you to understand the problem?				
DTf Description of the temporal proximity of events				

DTf13 If you had to order the actions that happened chronologically, how many actions would you count?
DTf14 At what point in the situation is it narrated: "A soldier watches the wall", at the beginning, in the middle or at the end of the situation?
DTf15 Do you consider it important to know what happened before and what happened after to understand the problem?
DCg Identification of any explicit or inferred causal relationship
DCg16 What generated the displacement of the soldier when he was in his initial position?
DCg17 Why did the soldier stop when he was walking to the right?
DCg18 Do you consider this information important to understand the problem?

To analyze the drawings, the classification of the drawings reported by Rellensmann et al. (2017) was taken as a reference. In this distinction between a situational model and a mathematical model, they point out that the former is more commonly generated by students with lower mathematical ability or some type of difficulty associated with understanding the problem and more specifically with the ability to build a model of suitable situation. A situational drawing is a low level of abstraction drawing where the objects are described pictorially, however, they do not disqualify their potential to successfully solve a problem that requires mathematical modeling, since it helps to deepen the understanding of the problem situation. In our analysis we consider the relevant elements of the situation that are represented in the drawing to create three categories of a situational drawing, which appear in Table 6.

Table 6: Categories of a situational drawing

Category	Characteristics	Code
Complete Situational Information	Represents all relevant elements (tokens, objectives, actions, description and spatial relationship and temporal proximity) based on the Event Indexing Model.	ISC
Partial Situational Information	Represents at least three relevant elements (chips or tokens, objectives, actions, description and spatial relationship and temporal proximity) based on the Event Indexing Model	ISP
Inconsistent Situational Information	Presents information related to a single relevant element of situational information (chips or tokens, objectives, actions, description and spatial relationship and temporal proximity) or does not represent related information.	ISI

Note: The codes presented correspond to the initials of each category

Results and Discussion

Based on the data collected, two major themes were constructed to answer the research questions. The following sections describe these topics.

Theme 1: Difficulties in following relevant elements of the SM prevent the successful resolution of Word Problems.

This theme is made up of the relevant elements of the *SM* that were omitted while reading the problems. The results show the difficulty in identifying tokens in written situations. The only token identified in the three problems was the protagonist. The intentionality of the protagonist was partially identified, despite that this knowledge is intuitive and immediate. In PV1 there was no clarity in the answer to the question asked for this indicator. The components of each Word Problem generated a high cognitive demand for the student. The following excerpt from the PV1 interview supports this interpretation:

I: What objects and characters are mentioned in the problem?
C: walks to the right, not to the left (stops). On the right side (secure)
I: Who is the protagonist?
C: The soldier, the wall (read the statement on the sheet again)
I: It's okay if you're not sure about your answer.
C: the meters

Consequently, in the three Word Problems, the description of a space determined by the context of the situation was a partially identified element, since not all the tokens were taken into account for an exact spatial distribution of the objects. The spatial

location was the essential and critical element that made it difficult to understand the problems. The interpretation is based on the following fragment taken from the interview to analyze PV2:

I: what is the starting position of the player?
C: the player is five steps behind the pins to launch the ball
I: In his starting position, does the player throw the ball directly?
C: the distance between him and the pins is 21 meters, and he throws the ball that slides on the 18-meter lane (read the problem pointing out this information)
I: In which direction did the throw of the bowling ball occur?
C: forward to throw the pins
I: What was the player's position after he released the ball?
C: 21 meters

An exact spatial distribution of the objects requires expressing the relationship between them and the protagonist through the use of numbers. That is why numbers are considered to fulfill this property of information and are significant to the extent that the location of all the relevant elements of space and their relationships are clearly identified. This explains that the answer to the question What was the player's position after he released the ball? and the answer 21 meters turns out to be incorrect. As has been reported, it is about making sense of numbers in order to use them usefully (Verschaffel et al., 2000). This phenomenon was complex for Carlos, so that in PV3, when asked what is the starting position of the diver? he did not consider giving an answer in terms of the height of the protagonist, but instead provided an answer based on common sense (he is on the springboard), but insufficient for solving the problem in terms of the meaningful use of numbers and mathematical abstraction (Leiss et al., 2010).

The temporal proximity of the events in the three problems was another omitted element. By omitting exact spatial information of all the relevant objects, difficulty was generated in the Temporal Dimension (as events unfold). For example, the following fragment of the interview represents the omission of the elevation of one meter on the springboard, prior to the fall into the pool indicated in PV3:

I: What is the starting position of the diver?
C: is on the trampoline
I: In his starting position, did the diver jump straight off the springboard?
C: yes
I: In which direction did the first act of the diver occur?
C: down, fell into the water

The above affected the construction of a coherent space-time framework. The most relevant element in the Causality dimension is to identify some explicit or inferred

causal relationship by the reader through their schemes. Carlos was not able to identify ideas that did not appear explicitly in the text, but that were required for its correct understanding. In PV1, the implicit idea was that the soldier's displacement to the right side of the door happened from his initial position (19 meters to the left of the door). This idea would lead Carlos to consider 35 meters to the right, until finally calculating (adding or subtracting) the final position of the soldier. In PV2, the implicit idea: the player had 3 meters of distance available before throwing the ball, equivalent to 5 steps, could not be identified by Carlos. Finally, in PV3, the idea that the swimmer submerged 2 meters in the pool because it was the number of meters, he climbed to reach the surface of the water, was not generated either.

The results demonstrate the complexity of taking into account both explicit and implicit information from the text (Silva, 2014). The omission of relevant elements of the spatiality dimension caused a lack of consistency in the temporality dimension, and together with the lack of inferences, they characterized an inadequate spatiotemporal framework and insufficient linkage with the knowledge of everyday life actions in each word problem.

A favorable result was the use of concrete materials in the interviews. These materials promoted interest, the verbalization of ideas and interpretations, and the simulation of the actions narrated in the text (Glenberg, Gutiérrez, Levin, Japuntich, & Kaschak, 2004). The words spoken quietly as self-explanations, the movement of the eyes towards the text, towards the hands and towards the objects reflected the effort to index each event in a coherent mental model. These interviews were the means to interpret Carlos's difficulties, and in turn, they promoted support to overcome his difficulties. Figures 2 to 4 show the manipulation of objects during the interviews.

Figure 2: Interview to analyze PV1

Figure 3: Interview to analyze PV2

Figure 4: Interview to analyze PV3

The interviews generated benefits for the case identified as a student with difficulty. Other results like these have been reported in interventions through clinical interview (Ginsburg, 1997) and therapeutic interview (Cohen and Manion, 1990).

Theme 2: *Situational drawings and their role in externalizing difficulties and*

improvements in the process of building Situation Models.

The primary data from Carlos's worksheets fell into the last category of Situational Drawing. The drawings corresponded to the drawing category with Inconsistent Situational Information (ISI), characterized by containing information related to a single relevant element. None of the drawings contained numbers to establish the relationship between the protagonist, the objects, and the space, as can be seen in Figure 5.

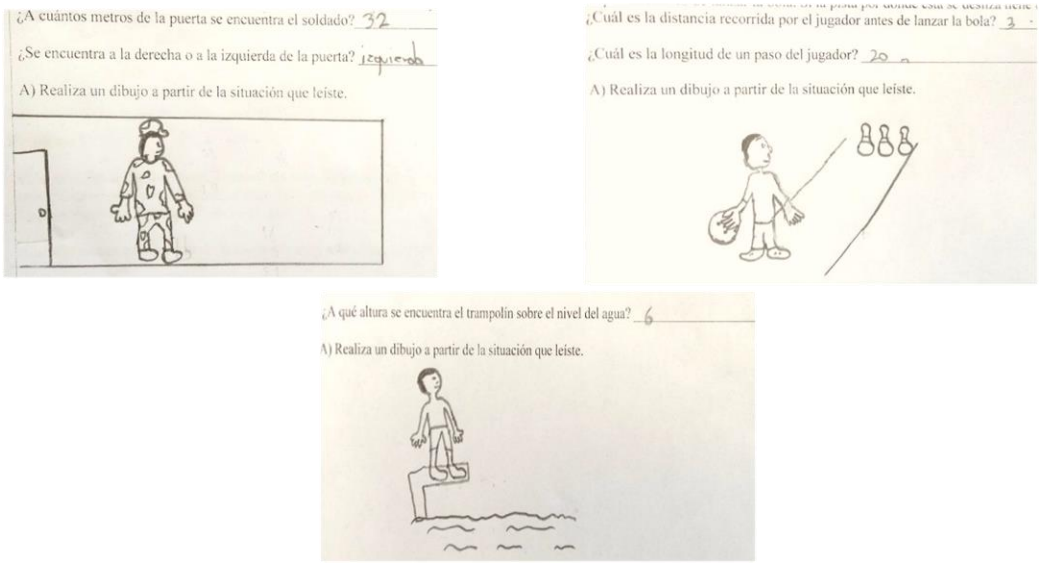

Figure 5: Carlos's initial responses and drawings in PV1, PV2 and PV3

In the drawing of PV1 the location of the gate in the center of the wall is not represented and, therefore, the left side of it is not shown either. In this problem, the location of the door right in the center of the wall represents the reference point to understand the protagonist's movement.

In PV2, the relevant object for understanding the problem was the location of the launch line. This element does not appear clearly in the drawing despite being mentioned in the first sentence. In an effort to solve the problem, he placed relevant elements such as the pins, the track, and the protagonist in his drawing; however, the throwing line was key to establishing relationships and properties (numbers) between the objects and the protagonist.

In PV3, although all the relevant elements were represented in the drawing according to the textual description of the problem (protagonist, diving board, water surface and the pool water), the spatial relationship between them was not established. That is, the presence of numbers is not shown to establish said relationship. In the

drawing in figure 3 the protagonist is seen in his initial position, however, according to the textual description of the problem, all the temporally contiguous events were not represented or expressed in numerical terms (lifting into the air, falling and immersion in the pool). The results obtained at the end of the interviews are presented in Figure 6.

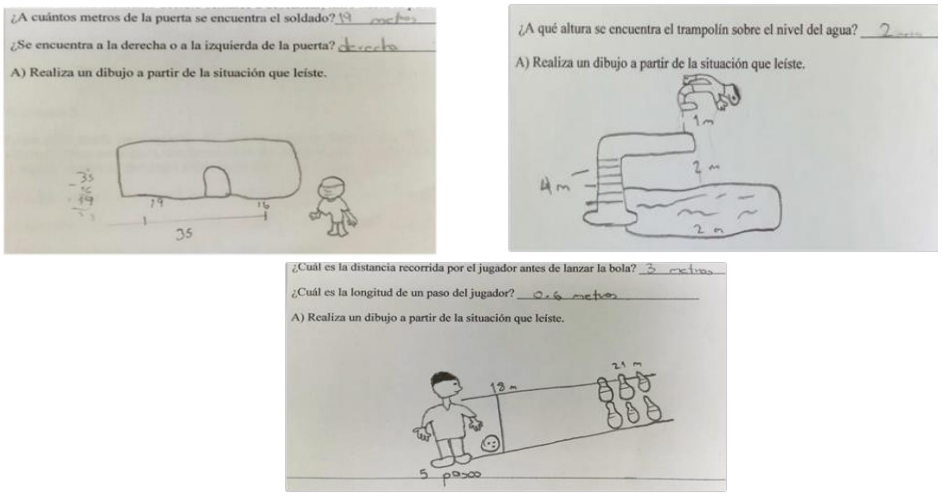

Figure 6: Responses and final drawings by Carlos in PV1, PV2 and PV3

These drawings presented Complete Situational Information (CSI), characterized by representing all the relevant elements (tokens, objectives, actions, description and spatial relationship and temporal proximity). In each drawing, the use of numbers was observed to represent the relationships between the protagonist, the relevant objects and the space. The meaningful use of numbers was promoted from the understanding of the problem and not from intuition or procedural mechanization. The answers generated in all three problems are correct. Greater attention was observed to the essential parts or relevant elements of each dimension to build coherent situation models.

In PV1 it was interesting to observe the phenomenon of immersion in the situation in order to adequately understand the problem (Juárez et al., 2014). Carlos adopted a different perspective from that of a spectator who draws and looks at the wall from the front, locating the right and left sides from a frontal perspective. In this problem, Carlos positioned himself as one more element of the situation, placing himself as the door or as one more element that served as a reference to locate the left and right sides in relation to himself. For this reason, when looking at the drawing made by Carlos from the front, the initial position and the final position are reversed. This result indicates the transformation of an isolated context situation reader to a context embedded situation reader.

Conclusions

The Event Indexing Model is a little-explored theoretical framework in Mathematics Education, however, its body of scientific knowledge on the textual comprehension process is abundant and has a wide field for empirical validation. This work used this theoretical perspective to specify the nature of the difficulties in the construction phase of Situation Models of three Mathematical Word Problems.

This work answers the research question by pointing out that the difficulties in the phase of construction of Situation Models during the resolution of Word Problems lie in the difficulty in identifying tokens relevant to the situation, difficulty in the construction of coherent spatio-temporal frames and difficulty generating inferences. These difficulties are described as the lack of ability to locate relevant elements of the space determined by the context of the situation and the establishment of properties (numbers) to establish the relationship between the protagonist, the objects and the space, as well as the difficulty in inferring the implicit causal relationships of the text.

Based on the evidence, we propose some suggestions for intervention and teaching in solving Word Problems with students with less developed linguistic and mathematical skills (Chin & Fu, 2021; Wijaya et al., 2019), such as the following:

1) Pose verbal problems for the mathematical activity that contain a good story (protagonist, intention, causality, spatio-temporal relationships, conflict and question or questions). In this way, *narrative rupture* would be avoided and *relevant and sufficient elements* for understanding would be available.
2) The use of concrete materials to generate coherent indexing of textual information with the events and situations that occur in the narrative of the problem (Glenberg , et al., 2004).
3) Promote immersion in the problem situation through the design of scenarios where the student with difficulty assumes the role of protagonist of the problem.
4) The use of situational drawings (Rellensmann et al., 2017) to represent tokens, objectives, actions, description and spatial relationship and temporal proximity. The development of a situational drawing is recommended to generate meaning for numbers in the comprehension phase and meaningful use in the solution phase. These types of drawings help the student to assign quantities as properties of the relationships between the relevant objects and protagonists of the situations in each Word Problem.

Studies with a larger sample size, a broader content approach, as well as the validation of the constructed interview questionnaires could contribute significantly to the design and implementation of interviews as an intervention to improve the textual understanding of Word Problems supported by the use of specific materials and drawings.

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